Collaborative Problem Solving in Senior Secondary Mathematics Classrooms

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This paper reports on research into patterns of social interaction associated with metacognitive activity in senior secondary school mathematics classrooms. Unsuccessful collaborative problem solving sessions were characterised by students' poor metacognitive decisions exacerbated by lack of critical engagement with each other's thinking, while successful outcomes were favoured if students challenged and discarded unhelpful ideas and actively endorsed useful strategies. The findings have practical significance for implementation of new mathematics curriculum policies that emphasise problem solving, mathematical reasoning, and communication.

In a review of progress in mathematical problem solving research in the 25 years to 1994, Lester (1994) lamented that research interest in this area appears to be on the decline, even though there remain many unresolved issues that deserve continued attention. One such issue highlighted by Lester was the role of metacognition in problem solving – where metacognition refers to what students know about their own thought processes, and how they monitor and regulate their thinking while working on mathematical tasks. Although the importance of metacognition is now widely acknowledged, we still lack an adequate theoretical model for explaining the mechanisms of self-monitoring and self-regulation, and understand too little about how metacognition and other aspects of thinking mathematically cohere to give individuals their mathematical "point of view" (Schoenfeld, 1992).

Also lacking are clear guidelines for teachers on how to foster higher order reasoning and problem solving skills, despite some research evidence that students can be taught to think mathematically. For example, Schoenfeld's work with college students emphasises the benefits of small group problem solving in developing both metacognitive control skills and a sense of what the discipline of mathematics is about. Beyond this work, however, research on small group learning in mathematics has yielded few insights into how students think and learn while interacting with peers, since most studies have focused on narrow learning *outcomes*, such as memorisation of facts or computational skills, rather than learning *processes* associated with mathematical reasoning (Good, Mulryan, & McCaslin, 1992). Consequently, the potential for small group work to develop students' mathematical thinking and problem solving abilities has remained largely unexplored, along with related issues concerning the teacher's role in orchestrating students' discussion and social interactions.

The concerns outlined above are reflected in Lester's (1994) call for further research on the role of the teacher in a problem centred classroom, and on teaching and learning processes for small groups and whole classes as well as individuals. As suggested by Lester's research agenda, there are practical as well as theoretical reasons for studying mathematical thinking. Issues of practical interest arise from new curricular trends that emphasise problem solving, mathematical reasoning, and communication (Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989). The impact of these trends is clearly seen in the Queensland Senior Mathematics Syllabuses (e.g., Board of Senior Secondary School Studies, 1992), where assessable objectives for students' learning now refer to Communication and Mathematical Applications as well as Mathematical Techniques. These significant curriculum reforms are intended to engage students more actively in the mathematical enterprise, yet are likely to require a major shift in the classroom practices of many mathematics teachers. How can changes to the social organisation of classrooms be justified when our understanding of the ways in which students' mathematical thinking is cultivated by these new forms of classroom interaction is far from complete? This is an important question that is neglected by curriculum documents promoting reform in mathematics education.

The challenges outlined above point to the need for further research on mathematical thinking and learning to be conducted in authentic classroom settings. This need was addressed by the study whose findings are presented here. The study investigated the characteristics of senior secondary students' metacognitive activity as they worked together on mathematical tasks, and considered how teachers can create a classroom culture of inquiry which promotes mathematical habits of mind (Goos, 2000). The specific research question addressed by this paper asks how metacognitive strategies are elicited and supported during collaborative peer interaction.

Collaborative Metacognitive Activity

Previous research on metacognitive development has used Vygotsky's notion of the zone of proximal development (ZPD) to explain how adults can scaffold learners' progress from assisted (other-regulated) to independent (self-regulated) performance (e.g., Bruner, 1985; Wertsch, 1985). It has also been suggested that interaction between peers with incomplete but roughly comparable expertise can create a collaborative ZPD in which students are able to coordinate their different perspectives in order to achieve progress (Forman & McPhail, 1993). However, the literature that deals specifically with improving metacognitive strategy use via peer interaction is limited, and has produced conflicting results (cf Artzt & Armour-Thomas, 1992; Forman & Cazden, 1985; Stacey, 1992). In particular, it is unwise to assume that students will interact spontaneously in productive ways, especially if they are not accustomed to working with peers as part of their regular classroom experience.

Although there is a large body of literature devoted to peer learning (see Good, Mulryan, & McCaslin for a review), not all forms of peer interaction can be classed as collaborative. For example, Damon and Phelps (1989) distinguish between various approaches to peer education according to the quality of engagement that is fostered. Thus they define "peer tutoring" as interaction in which students of unequal expertise are brought together so that one may instruct the other, and "cooperative learning" as an arrangement which allows teams of students to divide a task and master its separate parts. Damon and Phelps reserve the term "peer collaboration" for the interaction that occurs when students with similar levels of competence share their ideas in order to jointly solve a challenging problem. In this context of supportive communication and assistance students are encouraged to experiment with new ideas and critically re-examine their own assumptions – a form of interaction that seems to hold promise for improving students' metacognitive awareness and regulatory strategies.

Similarly, Granott (1993) maintains that highly collaborative interactions between peers of equal expertise are characterised by shared activity, a common goal, continuous communication, and co-construction of understanding. This view is consistent with the definition of collaboration offered by Teasley and Roschelle (1993), as "a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of the problem" (p. 235).

From the above the distinguishing feature of peer collaboration can be defined as *mutuality* – a reciprocal process of exploring each other's reasoning and viewpoints in order to construct a shared understanding of the task. Because this kind of reasoned dialogue involves comparing one's own ideas with those of another person, collaborative interaction need not be based purely upon agreement and cooperation, but may also include disagreement and conflict. Thus the process of co-constructing understanding is more complex than simply reaching consensus on an agreed answer (Kruger, 1993). The purpose of this paper is to characterise mechanisms of peer collaboration that contribute to shared understanding through metacognitive activity in problem solving tasks.

The Study

The research study was conducted in senior secondary school classrooms over a three year period (1994-96). Five teachers and their mathematics classes, all in different schools, contributed to the study. As the emphasis was on interpreting learning in complex social settings rather than experimental manipulation and control of variables, research methods were consistent with a naturalistic inquiry approach (Lincoln & Guba, 1985), and included long term participant observation of classrooms (supplemented by audio and video recording), interviews with students and teachers, and survey questionnaires. Complementary perspectives provided by questionnaire and observational data resulted in one classroom being selected for intensive analysis over both years of the study.

Target students were chosen for observation on the basis of their metacognitive sophistication and preference for working collaboratively with peers, as judged from preliminary observations and questionnaire responses (refer to Goos, 1995, for questionnaire details). One lesson was observed each week, and target students were videotaped and audiotaped as they worked together.

Data Coding and Analysis

Selected portions of audio and videotapes were transcribed and subjected to two coding passes focusing on the conversational turns of all speakers (referred to here as Moves). Moves in the protocol were first coded to identify their metacognitive function. A coding scheme developed in an earlier study (Goos & Galbraith, 1996) was used to identify metacognitive acts where a *New Idea* was proposed or an *Assessment* of particular aspects of the solution (i.e., strategy, result, understanding) was made.

Conversational Moves were then coded a second time to identify their contribution to the collaborative structure of the interaction, indicated by the *transactive* quality of the dialogue. Transactive reasoning is defined as clarification, elaboration, justification, and critique of one's own or one's partner's reasoning (Kruger, 1993). Three types of transacts were coded: spontaneously produced transactive *statements* and *questions*, and passive *responses* to transactive questions. The orientation of each transact was also noted: operations on one's partner's ideas were labelled other-oriented, while reasoning directed at one's own ideas was coded as self-oriented. This procedure produced six transact codes: (three types) × (two orientations). While Teasley (1997) has argued that transactive coding of peer discussion is consistent with Vygotskian approaches to studying links between collaborative interaction and cognitive change, this approach does not do justice to the reciprocal nature of collaboration. The scheme was therefore modified by grouping the codes as follows:

- *Self-disclosure* – Self-oriented statements and responses that clarify, elaborate, evaluate, or justify one's own thinking.
- *Feedback Request* – Self-oriented questions that invite a partner to critique one's own thinking.
- *Other-monitoring* – Other-oriented statements, questions and responses that represent an attempt to understand a partner's thinking.

Successful Collaboration

Three transcripts were selected for detailed analysis to illustrate common features of collaborative metacognitive activity. The transcripts were drawn from different years of the study (1994, 1995, 1996), and involve three different groups of students. All record students' interactions in the early stages of learning a new mathematical concept or process (compound interest, projectile motion, and Hooke's Law), so that the tasks on which they worked were unfamiliar, challenging, and genuine "problems". The tasks were therefore likely to require metacognitive control of problem solving actions, and to elicit collaborative interaction.

Separate analyses of the metacognitive and transactive nature of the dialogues were undertaken for each transcript. In addition, collaborative metacognitive activity was identified in Moves double coded both as metacognitive acts and transacts. A summary of metacognitive transacts for all three problem solving protocols is provided in Table 1.

Table 1
Moves Double Coded as Metacognitive Acts and Transacts

	Transactive Structure (Frequencies)		
Metacognitive Function	Self-disclosure	Feedback Request	Other-monitoring
New Idea	14	2	4
Assessment–strategy	2	5	16
Assessment–result	–	3	–
Assessment–understanding	–	–	6
Total	16	10	26

These results indicate that joint metacognitive activity was characterised by:

- students clarifying, elaborating and justifying their New Ideas for the benefit of a partner *(Self-disclosure)*;
- students asking their peers for help in finding errors by inviting critique of strategies and results; and students seeking feedback on the New Ideas they proposed *(Feedback Request)*;
- students making an effort to understand their partners' thinking by offering critiques of their strategies, requesting explanations, or elaborating on and monitoring their understanding of partners' ideas *(Other-monitoring)*.

For the three transcripts (a total of 351 Moves), 26 of the metacognitive-transacts were self-oriented (Self-disclosure or Feedback Request) and 26 were other-oriented (Other-monitoring). Thus, when the students interacted with each other, their monitoring activity was directed at both their own thinking and the ideas of their peers.

Unsuccessful Collaboration

This microscopic analysis of problem solving transcripts provided a lens through which to view the peer interaction processes that establish a collaborative zone of proximal development. Nevertheless, it is necessary also to examine situations in which collaboration was metacognitively fruitless, and to identify reasons for this lack of success.

A framework for classifying types of metacognitive *failure* was created to guide selection and analysis of unsuccessful problem solving episodes. The metaphors of *blindness*, *vandalism*, and *mirage* were chosen to describe situations where students respectively overlooked errors, applied inappropriate conceptual structures to resolve an impasse, and reacted to difficulties that did not exist (see Goos, 1998, for details). A further three transcripts of students' dialogue were selected as before to illustrate each of these scenarios, with the analysis showing how poor metacognitive decisions contributed to problem solving failure. (The problem solving tasks dealt with combinatorics, motion of a body on an inclined plane, and area of the Koch snowflake). However, a more detailed analysis was called for if some of the subtle processes distinguishing successful from unsuccessful collaborative metacognitive activity were to be revealed.

The Role of Transactive Discussion

The first step was to calculate proportions of "success" and "failure" transcripts coded as having metacognitive function and/or transactive structure. This analysis showed that there was very little difference in the metacognitive proportions recorded in both groups of transcripts. However, a different picture emerged from the summary of transact proportions: first, there was a lower incidence of transactive discussion in unsuccessful problem solving sessions (17% of all Moves, compared to 26% in successful sessions); and second, a large part of this discrepancy was accounted for by the difference in the proportions of *non*-metacognitive transacts (4.7% in unsuccessful problem solving sessions, compared to 11% in successful sessions). It appears, then, that success is characterised not only by utterances which are simultaneously metacognitive and transactive (the double-coding criterion applied previously), but also by interactions involving purely transactive discussion. This finding suggests that the discussion *around*, and generated by, individual metacognitive acts is crucial to the success of the mathematical enterprise.

All transcripts were re-examined to identify metacognitive acts that either led to or were prompted by a transactive statement, question, or response. Moves so identified were thus connected to at least one transact, and were labelled *metacognitive nodes*. If the node was connected to more than one transact, then a *transactive cluster* was said to have formed around the node. For example, when working on a compound interest spreadsheet problem one student (Rob) challenged his partner (Belinda) to justify her proposal that the interest rate for each compounding period was 1.01:

52. B: Point zero one. *(New Idea)* It's not actually point zero one, it's (inaudible). *(Self-oriented transactive statement, clarification of the New Idea)*
53. R: How did you work that *out*? *(Other-oriented transactive question, seeking justification of New Idea in Move 52)*
54. S: You divided the –
55. B: See, compound interest, you've got to add one to it. *(Self-oriented transactive response to Move 53, justification of New Idea in Move 52)*
56. R: Is that it? That it, one? What for? *(Other-oriented transactive question, seeking further justification of Moves 52/55)*

Here, Move 52 is a metacognitive node because it prompts the transactive interchange, or cluster, comprising Moves 53, 55 and 56. This may be represented visually as in Figure 1. Note that circles superimposed on node symbols represent metacognitive transacts (e.g., Move 52), while arrows connecting metacognitive nodes with transacts, and particularly with transactive clusters, pinpoint instances of extended discussion of metacognitive ideas and assessments, and thus highlight the role of *non*-metacognitive transacts.

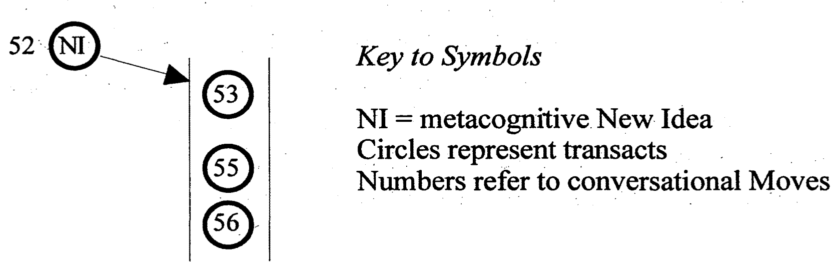

Figure 1. Visual representation of metacognitive nodes and transactive clusters.

A complete visual representation of nodes and clusters was constructed for each transcript, and the numbers and proportions of nodes and clusters for successful and unsuccessful problem solving transcripts were recorded (see Table 2).

Table 2
Frequencies and Proportions of Metacognitive Nodes and Transactive Clusters for Successful and Unsuccessful Collaborative Metacognitive Activity

	Frequencies (Proportions)			
	Success Transcripts		Failure Transcripts	
Metacognitive nodes	26	(0.074)	11	(0.040)
Transactive clusters	10	(0.028)	3	(0.011)
Total Moves	351		277	

Successful collaboration was found to feature roughly twice the proportion of metacognitive nodes and transactive clusters as unsuccessful collaboration. In other words, transactive discussion of metacognitive New Ideas and Assessments appears to be a significant factor in successful collaborative problem solving. Closer inspection of the dialogues revealed that the interplay between transactive *challenges* and metacognitive decisions was significant in shaping problem solving outcomes, since challenges eliciting clarification and justification of strategies stimulated further monitoring that led to errors being noticed or fruitful strategies being endorsed. On the other hand, causes of metacognitive failures could be traced to the absence of such challenges. Being held accountable by peers for explaining "how" and "why" may have prompted students to explore an idea more thoroughly, or to step back from a task and recognise a mistake or anomaly.

Conclusions and Implications

This study set out to examine metacognition in the context of senior secondary school classrooms in order to develop a theoretical and practical rationale for the type of mathematics teaching envisioned by the NCTM *Standards* (National Council of Teachers of Mathematics, 1989) and the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). Because the study of metacognition has its roots in cognitive psychology, research in this area has tended to treat monitoring and regulation as individual, in-the-head processes that liken metacognitive control to internalised self-instruction. The present study has extended this view of metacognition as self-directed dialogue to include collaborative conversations that made the processes of monitoring and regulation overt. The notion of a collaborative zone of proximal development established through interaction between peers of comparable expertise has previously received little attention from researchers interested in relationships between small group processes and mathematical thinking.

Interestingly, the study of metacognitive failure afforded significant insights into possible mechanisms for successful collaboration. Previous research on metacognitive aspects of individual students' mathematical thinking has suggested that failure is virtually guaranteed by poor metacognitive control decisions (e.g., Schoenfeld, 1992), and there is some evidence that these decisions can be adversely affected by peer interactions in small group problem solving (e.g., Stacey, 1992). The results presented here extend and qualify these findings by highlighting the significance of transactive discussion in making good metacognitive decisions.

Working together on problems offered a realistic context for students to verbalise their ideas so that their implications could be considered and evaluated. Nevertheless, it may be helpful for teachers of younger or less able students to provide rubrics, or key phrases, to explicitly encourage them to engage with each other's thinking. Consider, for example, a pair of students working together on a problem (call them Student A and Student B). There are four ways in which Student A's thinking may become the subject of discussion:

1. Spontaneously, and initiated by the student (Self-disclosure, Student A operates on Student A's thinking: *Here is my idea*).
2. Spontaneously, and initiated by the partner (Other-monitoring, Student B operates on Student A's thinking: *Here is what I think of your idea*).
3. Through an invitation issued by the student. (Feedback Request, Student B is asked to operate on Student A's thinking: *What do you think of my idea?*).
4. Through a partner's challenge. (Other-monitoring, Student B asks Student A to operate on Student A's thinking: *What do you mean?*).

The italicised phrases provide a useful template for teachers to structure their students' mathematical discussions so that ideas are shared and evaluated.

Even with guidelines for discussion, critics of group work claim that students' thinking can become confused without the teacher's guidance, and this observation raises questions as to whether, when, and how the teacher should intervene to redirect students' efforts to more productive ends. Teachers face a number of dilemmas in tailoring assistance to meet students' specific needs. For example, their intervention may be misdirected and cause more confusion than clarification, or may deny students the opportunity to resolve their own difficulties. Decisions also need to be made about the timing of teacher interventions, and, indeed, whether to intervene at all. Such decisions become complicated when groups and individuals are working on different tasks in any given lesson, including tasks not sanctioned by the teacher. There are finely tuned appraisals to be made about the timing, amount, and type of assistance to provide, if a delicate balance between encouraging persistence and avoiding frustration is to be maintained. Further research is warranted on the development of mathematical thinking in secondary school classrooms in order to pursue these and other practical questions arising from this study.

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